Lecture 6

Material and Homework

Read:
Chapter 5 Section 5.4, 5.6 Yariv and Yeh
Chapter 6 Sections 6.1-6.5 Yariv and Yeh
Appendices 1 and 2 in Coldren (on reserve)

Homework #3
Due Tuesday, May 2, 2006

a. Derive equation 5.4-1 in Yariv starting with equation 5.3-8
b. Problem 5.1 Yariv
c. Problem 5.6 Yariv

Dispersion and Complex Refractive Index

- Looking more closely at n from previous lecture (harmonic oscillator model),

$$n \approx 1 + \frac{\chi}{2} = 1 + \frac{Nq^2}{2m\varepsilon_0\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)}$$

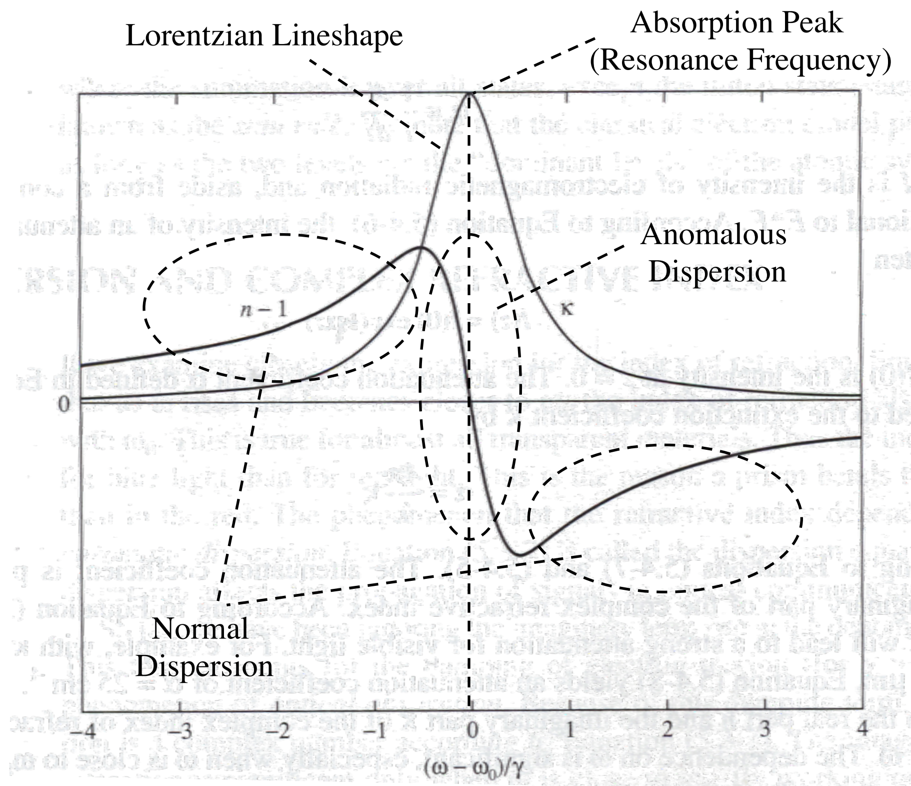
$$= 1 + \frac{Nq^2}{4\omega_0 m\varepsilon_0\left(\omega_0 - \omega + i\gamma / 2\right)}$$

- Using the notation for n as n' to denote a complex number, we see that as ω approaches ω_0 and assuming $\chi \ll 1$, the index of refraction increases. This change in refractive index as a function of optical frequency is called ***Chromatic Dispersion***.
- Looking at the imaginary part of n' we see that the optical E-field will be attenuated as ω approaches ω_0 giving rise to ***Optical Absorption***.

$$n' = n - i\kappa = 1 + \underbrace{\frac{Nq^2\left(\omega_0^2 - \omega^2\right)}{2m\varepsilon_0\left[\left(\omega_0^2 - \omega^2\right)^2 + \left(\gamma^2\omega^2\right)\right]}}_{\text{Chromatic Dispersion}} - i\underbrace{\frac{Nq^2\gamma\omega}{2m\varepsilon_0\left[\left(\omega_0^2 - \omega^2\right)^2 + \left(\gamma^2\omega^2\right)\right]}}_{\text{Absorption}}$$

Dispersion and Complex Refractive Index

- Plotting the real (n-1) and imaginary (κ) part of the complex refractive index as below, we see the classic relation between index and absorption (gain) for light interacation in atomic systems where are atoms are identical.
- We note the following characteristics:
  - For the imaginary part of the refractive index (κ) the center of the absorption peak is located at ω_0 and falls off symmetrically as ω increase and decreases.
  - $\kappa(\omega)$ has a ***Lorentzian*** lineshape
  - The real part of the refractive index (n) approaches unity at low frequencies ($\omega < \omega_0$) and increases and peaks as ω approaches ω_0. This region where ***$dn/d\omega > 0$*** is called ***Normal Dispersion***.
  - As ω gets close to ω_0 (very near resonance, where the absorption is high), the index decreases and passes through 1 with increasing frequency. This region where ***$dn/d\omega < 0$*** is called ***Anomalous Dispersion***
  - The relationship between the real (n-1) and imaginary (κ) part of the complex refractive index is dictated by the ***Kramers-Kronig*** relationship.



Sellmeir Equation

- If we are dealing with a material that has a collection of different types of atoms, wach with its own resonance frequency (ω_{oj}), damping factor (γ_j) and fractional per volume (A_j), then we can
  - Treat each one as having the absorption and index properties described by the harmonic oscillator model
  - Linearly superimpose the absorption and index profiles for each atom for the whole material

$$n^2 = 1 + \frac{Nq^2}{m\varepsilon_0} \sum_j \frac{A_j}{\left(\omega_j^2 - \omega^2 + i\gamma^2\omega\right)}$$

Absorption and Amplification

- For a 2-level atomic system, we define the following transition rates assuming the induced rates from levels 1 to 2 and 2 to 1 are proportional to the photon densities (per unit frequency)

$$\left(W_{21}^{'}\right)_{induced} = B_{21}\rho(\nu)$$
$$\left(W_{12}^{'}\right)_{induced} = B_{12}\rho(\nu)$$

- The total downward rate (2 to 1) is the sum of the induced and spontaneous rates

$$W_{21}^{'} = B_{21}\rho(\nu) + A_{21}$$
$$W_{12}^{'} = \left(W_{12}^{'}\right)_{induced} = B_{21}\rho(\nu)$$

- We also assume that the atoms are blackbody radiators in thermal equilibrium at temperature T with spectral density

$$\rho(\nu) = \frac{8\pi n^3 h\nu^3}{c^3}\frac{1}{e^{h\nu/k_B T} - 1}$$

Einstein Relations

- In thermal equilibrium, the number of 2 to 1 and 1 to 2 transition rates are balanced

$$N_2 W_{21}' = N_1 W_{12}'$$

$$N_2 \left(B_{21} \frac{8\pi n^3 h\nu^3}{c^3} \frac{1}{e^{h\nu / k_B T} - 1} + A_{21} \right) = N_1 \left(B_{12} \frac{8\pi n^3 h\nu^3}{c^3} \frac{1}{e^{h\nu / k_B T} - 1} \right)$$

- The distribution of energy states for atoms in TE for a two level system will follow the Boltzman distribution

$$\frac{N_2}{N_1} = e^{-h\nu / k_B T}$$

- Equating N_2/N_1 above, we see that the following relations (Einstein relations) must be satisfied

$$B_{12} = B_{21}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}$$

Energy Density

- Defining the spontaneous lifetime by $t_{spont}=1/A_{21}$, we can write the transition rate per atom due to an incident optical field with a uniform spectrum and energy density $\rho(\nu)$, beam intensity $I_0=cU_0/n$ (watts per square meter), U_0 is the energy density of a monochromatic field

$$W_i' = \frac{A_{21}c^3}{8\pi n^3 h\nu^3}\rho(\nu)$$

$$= \frac{c^3 U_0}{8\pi n^3 h\nu^3 t_{spont}} g(\nu)$$

$$= \frac{\lambda^2 I_0}{8\pi n^2 h\nu t_{spont}} g(\nu)$$